The CALPHAD method – basis and applications



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- 2. General idea behind CALPHAD
- 3. Thermodynamic basis
- 4. Thermodynamic models
- 5. Some Examples:
- 6. Extension of CALPHAD beyond thermochemistry
- 7. Conclusions





1. Introduction - A shift in Paradigm!

New possibilities: Development time for new materials can be decreased from 10-20 years to 3-4 years.



Method for a purpose

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CALPHAD - the first materials genome because it is ...

- the most efficient way of integrating various pieces of information
 - different character (thermochemistry, phase diagrams etc)
 - coherent and useful form
- extendable far beyond the traditional thermochemistry
- increasingly being used outside the traditional CALPHAD community

A major enabling technology in Materials science and engineering.





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CALPHAD – a brief history



- Was born 1970 with the book (Kaufman and Bernstein: Computer calculation of phase diagrams, 1970)
- In the 50: ies and 60: ies Kaufman and Cohen and Hillert had outlined the general principles.
- Early predecessors van Laar 1908 and later several others.
- CALPHAD was introduced as an alternative to the quantumbased approach PHACOMP (1964).
- The annual meetings (Later called CALPHAD conferences) and the CALPHAD journal both initiated by Kaufman were instrumental for the rapid expansion of the field.





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2. General idea behind CALPHAD

Properties such as

- Phase equilibrium
 - Phase diagrams
 - Composition of phases and compounds
 - Partition coefficients and equilibrium constants...
- Thermochemical data
 - Heat of reactions and transformations
 - Heat capacities
 - Vapour pressures...
- Elastic properties
 - Bulk modulus
 - Elastic constants...
- Volume and its thermal expansion...

all stem from a single thermodynamic function of the system of interest.

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If that function is known...

then

- all these properties may be calculated using that function.
- also other quantities of interest may be calculated such as
 - Driving forces for reactions and transformations if the system is not at equilibrium (e.g. In phase-field simulations)
 - Properties of metastable states, e.g. metastable phase diagrams.





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3. Thermodynamic basis

- What is the equilibrium state of a system aged under given external conditions?
- What are the driving forces for internal changes if the system is not in equilibrium?





Important remark:

Thermodynamics <u>does not only</u> apply at equilibrium, i.e. the condition when nothing more can change.

- measurements can be performed sometimes quite far outside equilibrium and be extrapolated further into the non-equilibrium regime.
- Thus thermodynamics applies considerably outside equilibrium. But how far outside would it apply? Here we will demonstrate how thermodynamics may be applied as far as needed to solve certain problems.





Overview of thermodynamics



First law: Energy, heat and work

Second law: Entropy, equilibrium, driving force

Multicomponent systems -chemical potential





Surroundings External conditions: *P*, *V*, *T*, n_k ...



Interactions between system and surroundings:

composition C thermal 1 mechanical 1 C+2

After fixing the external conditions the system will approach a <u>state</u> <u>of equilibrium</u> at which there are no further changes.





C + 2 external variables must be fixed in order to define a unique equilibrium state.

The equilibrium state may be represented by a point in a C + 2 dimensional space; a <u>state diagram.</u>

A state diagram containing information about what phase is stable is a **phase diagram**.





Example:

1-component system, n = fix, let *P* and *T* vary =>

2-dimensional state diagram.

Add information on phases => unary phase diagram

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Example: Phase diagram of pure iron. Pressure in Pa and temperature in K.





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External and internal state variables

External state variables

may be controlled outside the system by the experimentalist or the processing conditions

Internal state variables

In a system out of equilibrium additional variables are needed to fully characterize the system. These variables

- describe the internal constitution of the system
- vary until they reach their equilibrium values and the system comes to rest





Intensive variables

If a new system is formed by merging two identical systems the values of all intensive variables remain constant. Ex: *P*, *T*, ρ , *c*_k, *H*_m etc

Extensive variables

If a new systems is formed by merging two identical systems the values of all extensive variables will become twice as large.

Ex: *V, U, n_k, Additative rule:*

$$V = V_1 + V_2$$





Two types of intensive variables:

- Potentials ex: *P, T*
- Densities or specific variables =

extensive variable size

$$\rho = \frac{\text{mass}}{\text{volume}} \quad c_k = \frac{n_k}{\text{volume}} \quad x_k = \frac{n_k}{\sum n_j} \quad \dots$$

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The state of equilibrium at given pressure, temperature and composition

Equilibrium for the value of the internal variable ξ that gives the lowest Gibbs energy.



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Change in Gibbs energy during a phase transformation.





a) Metastable equilibrium b) Unstable equilibrium (critical state).

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Combined 1:st and 2:nd law of thermodynamics in an open system (i.e. Exchange of matter with surroundings), i.e. multicomponent

$$dU = TdS - PdV + \sum_{k} \mu_{k} dN_{k} - Td_{i}S \quad \text{(combined law)}$$

$$Td_{i}S = \sum_{j} D_{j}d\xi_{j} \ge 0 \quad (2: \text{nd law})$$

$$D_{j} = \text{driving force for process } j$$

$$d\xi_{j} = 0 \quad \text{Process } j \text{ is frozen in}$$

$$U = U(S, V, N_{k}, \xi_{j})$$

$$S, V, N_{k}, \xi_{j} \quad \text{are natural variables for } U!$$

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From the combined law:

Each derivative tells how much the internal energy changes per unit amount of an extensive quantity, i.e. entropy, volume or number of moles of *k*, added to the system: Potentials!

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Equilibrium conditions come from second law:

For a closed system with a constant energy dU = 0

under constant volume/

$$dU = TdS - PdV + \sum_{k} \mu_{k} dN_{k} - Td_{i}S$$

The condition $dS = d_i S > 0$ implies that *S* must increase towards a maximum which is reached at equilibrium.



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A more convenient equilibrium condition

Equilibrium conditions fixed *T* and *P* we obtain from the combined law: $dU = TdS - PdV - D d\xi$ $d(U - TS + PV) = -SdT + VdP - D d\xi$ At constant *T* and $P: d(U - TS + PV) = -D d\xi$ G = U - TS + PV Gibbs energy $\left(\frac{\partial G}{\partial \xi}\right)_{TP} = -D = 0$ at equilibrium.

At given *T* and *P* the Gibbs energy *G* is minimized at equilibrium.

Most useful!





Fixed	Function to be minimized	Name
variables		
U, V, N _k	-S	Negative entropy
S, V, N _k	U	Internal energy
T, V, N_k	F=U-TS	Helmholtz energy
T, P, N_k	G=U-TS+PV=F+PV	Gibbs energy
S,P, N_k	H=U+PV	Enthalpy
<i>U</i> , <i>V</i> , μ _k	$-PV=U-\sum N_k \mu_k$	Grand potential
<i>T</i> , <i>P</i> , $N_k \mu_j$	$G-\sum N_j\mu_j$	





Characteristic state functions with their natural variables

 $U(S,V,\xi)$ $F(T,V,\xi)$ $G(T,P,\xi)$ $H(S,P,\xi)$ $S(U,V,\xi)$

Either one of these functions fully characterize a system.





If $G_m(T, P, x_1, x_2...x_n)$ is known, what is μ_j ?

It is straight forward to show that

$$\mu_{j} = G_{m} + \frac{\partial G_{m}}{\partial x_{j}} - \sum_{i=1}^{n} x_{i} \frac{\partial G_{m}}{\partial x_{i}}$$



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Example: Binary system – The tangent construction

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Some properties derived from Gibbs energy of a phase:

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$$G_{m} = G_{m}(T, P, x_{1}, x_{2}...\xi_{1}, \xi_{2}...)$$

$$V_{m} = \partial G_{m} / \partial P$$

$$S_{m} = \partial G_{m} / \partial T$$

$$\alpha = (\partial^{2}G_{m} / \partial P \partial T) / \partial G_{m} / \partial P$$

$$c_{p} = -T\partial^{2}G_{m} / \partial T^{2}$$

• • •

$$\mu_{k} = G_{m} + \frac{\partial G_{m}}{\partial x_{k}} - \sum x_{j} \frac{\partial G_{m}}{\partial x_{j}}$$
$$D_{j} = -\frac{\partial G_{m}}{\partial \xi_{j}}$$
$$G = \sum_{\alpha} N^{\alpha} G_{m}^{\alpha} (T, P, x_{1}^{\alpha}, x_{2}^{\alpha} \dots \xi_{1}^{\alpha}, \xi_{2}^{\alpha} \dots)$$





Driving force under fixed P, T and composition

Combined law yields:

$$dG = -\sum D_j d \xi_j$$

Consider now a process where the ξ variables change from an initial state to a final state.

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The driving forces may vary in a complicated way during the integration the integral

$$\int dG = G^{final} - G^{start} = -\sum_{start} \int_{start}^{final} D_j d\xi_j$$

The integrated driving force.





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4. Thermodynamic models If we know Gibbs energy as a function $G_m = G_m(P,T,x_1,x_2,...\xi_1,\xi_2,...)$

for the individual phases in a system:

- We may calculate a lot of properties of practical interest.
- Calculate equilibrium state and phase diagram by minimizing G.
- Calculate driving forces to use in kinetic simulations.





Modeling of solutions

Gibbs energy per mole for a solution phase is normally divided in:

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Regular-solution type of models

$${}^{E}G_{m} = \sum_{i>j} \sum_{j} x_{i} x_{j} L_{ij}$$

where

$$L_{ij} = L_{ij} + \sum_{k} (x_i - x_j)^{k-k} L_{ij} = L_{ij} + (x_i - x_j)^{1-1} L_{ij} + (x_i - x_j)^{2-2} L_{ij} + \dots$$

Redlich-Kister polynomial
 $k = 0$: Regular solution
 $k = 1$: Sub-regular solution

k = 2: Sub-sub-regular solution

etc







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Phases with sublattices

$$(A_1A_2...)_{a_1}(B_1B_2...)_{a_2}....(...)_{a_t}....$$

For example:
Oxides, e.g. $(Ni, Mn)(O, N)$
Carbo-nitrides, e.g. $(Cr, Fe, Mo)_{23}(C, N)_6$
Interstitials, e.g. $(Fe, Mn, Si...)(C, Va)$
Salts, e.g. $(Na^{+1}, K^{+1})(Cl^{-1}, F^{-1})$







> The site fraction y_j^t is the fraction of lattice sites on sublattice t that are occupied by component k. $y_j^t = N_k^t / \sum_j N_j^t$

N = number of formula units

 $G_m = G / N$

One can usually not calculate the chemical potential of a component because the derivative $\partial G / \partial N_k$ cannot be taken without violating the constraints.

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In general, $(A_{1}, A_{2},...)_{a_{1}}(B_{1}, B_{2},...)_{a_{2}}(C_{1}, C_{2},...)_{a_{3}}...$ $\mu_{I} = G_{m} + \sum_{t} \frac{\partial G_{m}}{\partial y_{I_{t}}^{t}} - \sum_{t} \sum_{k} y_{k}^{t} \frac{\partial G_{m}}{\partial y_{k}^{t}}$

where *I* is a hypothetical compound: $I = (I_1)_{a_1}(I_2)_{a_2}(I_3)_{a_3}...$ eg $Cr_{23}C_6$





The compound energy formalism

For example: how to represent the Gibbs energy of $(Cr, Fe)_{23}(C, N)_6$?

 G_m per mole of formula unit.

Random mixture on each sublattice

 $-S_m^{mix}/R = 23(y_{Cr} \ln y_{Cr} + y_{Fe} \ln y_{Fe}) + 6(y_C \ln y_C + y_N \ln y_N)$ What about the reference and excess energies? Note: in the reference mixing entropy and excess energy vanish.





The reference energy "surface"

4 compounds:

 $(Cr)_{23}(C)_{6} (Cr)_{23}(N)_{6}$ $(Fe)_{23}(C)_{6} (Fe)_{23}(N)_{6}$ $G_{m}^{ref} = y_{Cr} y_{C} \,^{\circ} G_{m}^{Cr_{23}C_{6}} + y_{Cr} y_{N} \,^{\circ} G_{m}^{Cr_{23}N_{6}} + y_{Fe} y_{C} \,^{\circ} G_{m}^{Fe_{23}C_{6}} + y_{Fe} y_{N} \,^{\circ} G_{m}^{Fe_{23}N_{6}}$

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The chemical potentials

$$\mu_{Cr_{23}C_6} = \left[G_m + \frac{\partial G_m}{\partial y_{Cr}^1} + \frac{\partial G_m}{\partial y_C^2} - \sum_t \sum_k y_k^t \frac{\partial G_m}{\partial y_k^t} \right]$$
$$\mu_{Cr_{23}C_6} = G_m^{Cr_{23}C_6} + y_{Fe} y_N \Delta^\circ G_{CrN+FeC} + RT \left[23\ln y_{Cr} + 6\ln y_C \right] + \dots$$

$$\Delta^{\circ}G_{CrN+FeC} = {}^{\circ}G_{m}^{Cr_{23}N_{6}} + {}^{\circ}G_{m}^{Fe_{23}C_{6}} - {}^{\circ}G_{m}^{Cr_{23}C_{6}} - {}^{\circ}G_{m}^{Fe_{23}N_{6}})$$





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5. Some examples





EXPERIMENTAL % AUSTENITE

• Longbottom and Hayes (1994) Nystrom and Karlsson (1994) × Gysel and Schenk (1991)

□ Longbottom and Hayes (1991)

Thorvaldsson et al. (1985)

• Maehera et al. (1983)

d < 4%

Average deviation

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Figure 10.40 Comparison between calculated and experimentally observed % of austenite in duplex stainless steels. (Data from Longbottom and Hayes (1991) represent dual phase steels.).

From: Saunders & Miedownik: "Calphad -a comprehensive review"



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Nominal composition (SAF 2507):

Fe – 25% Cr – 7% Ni – 4% Mo – 0.27% N – 0.02% C

Predict the temperature when sigma-phase becomes stable within some composition variation:

- FeBase
- Cr 23 27%
- Ni 6 8%
- Mo 3 5%
- N 0.25 0.29%
- C 0 0.03%



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The data challenge (ex. Ni-base alloys)

	Al	B	C	Co	Cr	Fe	Hf	Mo	Ν	Nb	Ni	Pd	Pt	Re	Si	Ta	Ti	V	W
В	X																()))		())()
С	X	X																	()))
Co	X	X	X																())()
Cr	X	X	X	X															
Fe	X	X	X	X	X	\cdots													()))
Hf	X	X	X	X	X	X											())))		
Mo	X	X	X	X	X	X	X												$\langle \rangle$
Ν	X	X		X	X	X		X	()))						()))		())))		()))
Nb	X	X	X	X	X	X	X	X	X										
Ni	X	X	X	X	X	X	X	X	X	X									$\langle \rangle \rangle \rangle$
Pd	X	X	X	X	X	X	X	X		X	X								
Pt	X	X	X	X	X	X	X			X	X	X							())
Re	X	X	X	X	X	X	X	X		X	X	X	X						()))
Si	X	X	X	X	X	X	X	X	X	X	X	X	X	X	())))				
Та	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X		\dots		\dots
Ti	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X		()))	()()
V	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	()()	())))
W	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	()()
Zr	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

- □ 20 elements: 190 binary systems
- □ 184 of the 190 binary systems assessed for full range composition
- All Ni containing ternaries plus other ternary systems also assessed to full range of composition (184 in total)
- 292 intermetallic and solution phases





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6. Extension of CALPHAD beyond thermochemistry

- Diffusion and phase transformations: driving forces from CALPHAD thermodynamics
- Other properties
 - Diffusion coefficients
 - Interfacial mobilities
 - Interfacial energies
 - Stacking fault energy
 - Elastic constants (anisotropic behaviour)
 - Optical
 - Electronic
 - Magnetic
 - Strength, ductility etc...





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From atoms to microstructure: The aim is to predict microstructure evolution and materials properties.



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Genomic database for diffusion

- Multicomponent systems: many diffusion coefficients!
- Various type of coupling effects may make it more complicated than Fick's law.
- A CALPHAD-type of approach was suggested for information on diffusion kinetics (Andersson-Ågren 1992)
 - Allowed systematic representation of the kinetic behaviour of multicomponent alloy systems.
- DICTRA was developed in the 1990s for numerical solution of multicomponent diffusion problems in simple geometries.





Diffusion in Ni-base alloys: Campbell et al. 2002

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(continued on next page)

Simple substitutional model

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Growth of oxides controlled by diffusion

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- Oxygen diffusion in the oxide layer gives inward growth
- Metal diffusion in the oxide layer gives outward growth
- Internal oxidation needs oxygen diffusion into the metal, i.e.
 Oxygen diffusion through the oxide scale and the metal

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Defect based models of diffusion in oxides

Vacancy mechanism operative on different sublattices. The defect structure, the vcancy content, calculated from CALPHAD databases, and the mobility parameters are stored in mobility databases.

Generalization the Wagner model!

Account for type A grain-boundary diffusion.





Experimental data on Fe tracer diffusion in spinel -Optimization of Fe mobilities

(Hallström et al. 2011)



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Simulation of oxidation of iron using the homogenization model in DICTRA





Mechanical properties

 Solution hardening (use the "compound energy formalism"):

Formula unit: $(M_1M_2...)(C, N, Va)_b$

Yield stress

$$\sigma_{y} = \sum_{ij} y_{i}^{'} y_{j}^{''} \sigma_{yij}^{\circ} + \Delta \sigma_{y}^{SSH}$$
$$\Delta \sigma_{y}^{SSH} = \sum_{ij} \sum_{k} (y_{i}^{'} y_{j}^{'})^{n} y_{k}^{''} A_{ijk} + \sum_{i} \sum_{k\ell} y_{i}^{'} (y_{k}^{''} y_{\ell}^{''})^{m} A_{ik\ell}$$

In classical models

n = m = 2/3

and the A parameters represent a combination of mismatch in lattice parameter and elastic constants. Here they are taken as adjustable parameters!

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7. Conclusive remarks

- Models in CALPHAD can have <u>any level</u> of sophistication and account for:
 - Crystallographic structure
 - Lattice vibrations
 - Chemical order and disorder
 - Other phenomena such as magnetic order disorder...
- Different phases in the same material can be represented by different models and require different type of parameters.
- Quantities which are experimentally unknown, uncertain or show a large scatter may be calcaluated by ab-initio methods.

CALPHAD is the most efficient method to organise our experimental and theoretical knowledge on thermodynamics and phase equilibria.

